

5 Замены переменных

Решите уравнения и системы:

1) $x^2 - 3x - 4 + 2\sqrt{x^2 - 3x + 11} = 0;$

2) $\sqrt[4]{x} + \sqrt{x} = 2;$

3) $2x^3 + \frac{5}{x^3} + 3 = 0;$

4) $x^2 + 3x + 4 + \frac{3}{x} + \frac{1}{x^2} = 0;$

5) $(x - 2)(x - 3)^2(x - 4) = 20;$

6) $\frac{16}{(x + 6)(x - 1)} - \frac{20}{(x + 2)(x + 3)} = 1;$

7) $\frac{24x}{2x^2 - 3x + 4} = \frac{12x}{x^2 + x + 2} + 5;$

8) $(2x^2 - 3x + 1)(2x^2 + 5x + 1) = 9x^2;$

9) $\begin{cases} \frac{3}{x} - \frac{4}{y} = 1, \\ \frac{2}{x} + \frac{5}{y} = 4.5; \end{cases}$

10) $\begin{cases} \frac{x - 1}{2x} + \frac{y + 1}{3y} = \frac{1}{4}, \\ \frac{3}{x} - \frac{2}{y} = \frac{7}{2}; \end{cases}$

11) $\begin{cases} \frac{1}{x + y} + \frac{1}{x + z} = \frac{7}{12}, \\ \frac{1}{x + y} + \frac{1}{y + z} = \frac{8}{15}, \\ \frac{1}{y + z} + \frac{1}{x + z} = \frac{9}{20}; \end{cases}$

12) $\begin{cases} x + xy + y = 5, \\ x^2 + xy + y^2 = 7; \end{cases}$

13) $\begin{cases} \frac{11}{2x - 3y} + \frac{18}{3x - 2y} = 13, \\ 48x - 77y - 6(x^2 + y^2) = -13xy; \end{cases}$

14) $(x^2 - 2x)^2 - (a + 2)(x^2 - 2x) + 3a - 3 = 0;$

15) $x^4 + (5 - 2a)x^3 - 8x^2 + (5a - 2a^2)x + a^2 = 0;$

16) $x^4 - 11x^3 - 10x^2 - 11x + 1 = 0;$

17) $\begin{cases} x^4 + 2x^2y^2 + y^4 + x + y = 28, \\ x^2 + y^2 - 2x - 2y = -1; \end{cases}$

18) $\begin{cases} 2\left(\frac{x^2}{y^2} + \frac{y^2}{x^2}\right) - 9\left(\frac{x}{y} + \frac{y}{x}\right) + 14 = 0, \\ x^2 + y^2 = 5; \end{cases}$

19) $\begin{cases} x^2(b^2 + 4b + 4) + xy(2b + 4) + y^2 = 9, \\ 2x - (b - 2)(x + y) = 6; \end{cases}$

20) $\begin{cases} \frac{1}{x + 1} - \frac{1}{y + 1} = \frac{2}{a}, \\ xy + x + y = -a^2 - 1; \end{cases}$

21) $(x + 1)(x + 2)(x + 3)(x + 6) = 36x^2.$